

ON-CENTER AND OFF-CENTER CELL COMPETITION GENERATES ORIENTED RECEPTIVE FIELDS FROM NON-ORIENTED STIMULI IN KOHONEN'S SELF-ORGANIZING MAP

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Summary

INTRODUCTION

The self-organization of sensotopic maps, in particular of visual maps, continues to be an area of great interest in computational neuroscience. In order to distinguish between the different map formation models and between the specific self-organization mechanisms they assume, their behavior with regard to an as large number of physiological, anatomical or theoretical constraints as possible has to be investigated. An interesting case in point is the development of oriented receptive fields from stimulus distributions or stimuli with rotational symmetry, i.e., without orientation. This physiologically quite plausible

symmetry breaking phenomenon has been observed in several models for the self-organization of receptive fields [1] or orientation maps [2, 3], which assumed a competition of On-center and Off-center cells, with rotational symmetry of the stimulus autocorrelation function. Whereas these models are characterized by a linear kernel operating on the afferent activity distribution and mediating the lateral interaction, a major competing model, Kohonen's Self-Organizing Map (SOM, [4]), employs a strongly non-linear lateral interaction function. This nonlinearity is presumably responsible for the successful reproduction of several properties of visual maps in respective SOM-models, like the widening of ocular dominance bands as a consequence of decreased stimulus correlation [5, 6], or the preferred angle of intersection between iso-orientation and iso-ocularity bands [7, 8]. However, the investigation of SOM-models with regard to the formation of oriented receptive fields upon stimulation with non-oriented On-center and Off-center stimuli turned out to be difficult, not the least because these models are numerically expensive to simulate.

The recent development of an analytical method to solve (simplified) SOM models (see accompanying paper) now allowed us to calculate conditions for the above mentioned symmetry breaking phenomenon in a SOM. We then used these results as a guiding line for the choice of parameters for simulations. In the remainder of this summary we give a brief sketch of the model, of the analytical results and of first numerical results.

METHODS

Self-Organizing Maps (SOM)

A Self-Organizing Map (SOM) consists of neurons characterized by a position \mathbf{r} in the map lattice plus a receptive field $\mathbf{w}_{\mathbf{r}}$. A stimulus \mathbf{v} is mapped onto that neuron \mathbf{s} whose receptive field $\mathbf{w}_{\mathbf{s}}$ matches \mathbf{v} best,

$$\mathbf{s} = \arg \max_{\mathbf{r}} (\mathbf{w}_{\mathbf{r}} \cdot \mathbf{v}). \quad (1)$$

This amounts to a winner-take-all rule, i.e. a strong lateral nonlinearity which can be regarded as a consequence of lateral inhibition [4]. The map results as a stationary state of a self-organization process, which successively changes all receptive fields $\mathbf{w}_{\mathbf{r}}$,

$$\Delta \mathbf{w}_{\mathbf{r}} = \epsilon h_{\mathbf{rs}} (\mathbf{v} - \mathbf{w}_{\mathbf{r}}), \quad (2)$$

following the presentation of stimuli \mathbf{v} . Here, ϵ controls the size of learning steps, $h_{\mathbf{rs}}$ denotes a neighborhood function, centered around the winning neuron \mathbf{s} and usually chosen to be of Gaussian shape,

$$h_{\mathbf{rs}} = e^{-\frac{\|\mathbf{r}-\mathbf{s}\|^2}{2\sigma^2}}. \quad (3)$$

h_{rs} enforces neighboring neurons to align their receptive fields. In this way the property of topography is imposed on the SOM.

The model

The set-up of our present model (Fig. 1) is very close to the set-up of the competing model by Miller [2] (the activation and adaptation rules are quite different, though!).

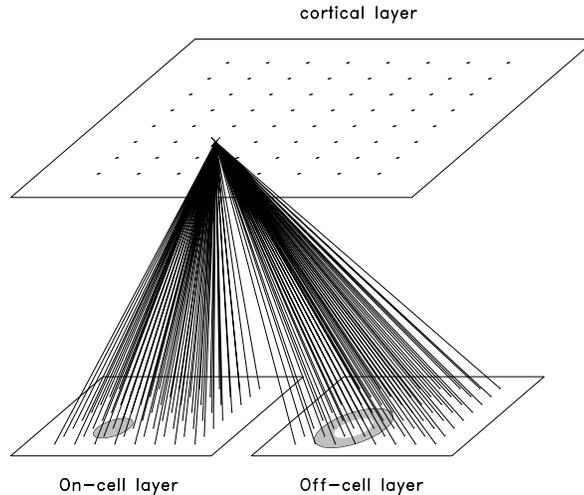


Figure 1: A cartoon of the model. Two two-dimensional input layers, modeling On- and Off-center afferents, resp., feed to a two-dimensional cortical layer. The model is fully connected, ie., each cortical neuron receives input from all On-center and Off-center neurons. The gray areas indicate an exemplary stimulus, in this case with a central peak in the On-cell layer, and an annulus in the Off-cell layer.

The activity distribution in the On-center and Off-center layers is assumed to result from spontaneous localized activity in retinal ganglion cells and is modeled as a peak of excitation in one of the layers (say, the On-center layer as in Fig. 1), complemented by an annulus-shaped activity distribution in the other layer. Peak and annulus result from the positive and negative parts of a “difference of two Gaussians” Mexican hat function (widths σ_1 , σ_2 , relative amplitude k of negative Gaussian). Position and sign (peak in the On-center or Off-center layer) of the stimuli are chosen at random. Note that the SOM-model in the present high-dimensional formulation does not require the a priori extraction of certain stimulus and receptive field features, as does the low-dimensional Self-Organizing *Feature Map* (SOFM) variant.

RESULTS

Analysis of the simplified model

An analysis of high-dimensional SOM-models like the present one can be performed using a distortion measure [6],

$$E_{\mathbf{v}} = \sum_{\mathbf{r}} \sum_{\mathbf{r}'} \sum_{\mathbf{v}' \in \Omega_{\mathbf{r}'}} \sum_{\mathbf{v} \in \Omega_{\mathbf{r}}} (\mathbf{v}' - \mathbf{v})^2 e^{-\frac{\|\mathbf{r} - \mathbf{r}'\|^2}{2\sigma^2}}, \quad (4)$$

which evaluates different possible states of a map via the different ways the stimuli are distributed among the neurons in these states ($\Omega_{\mathbf{r}}$ denotes the subset of stimuli which are mapped onto node \mathbf{r}).

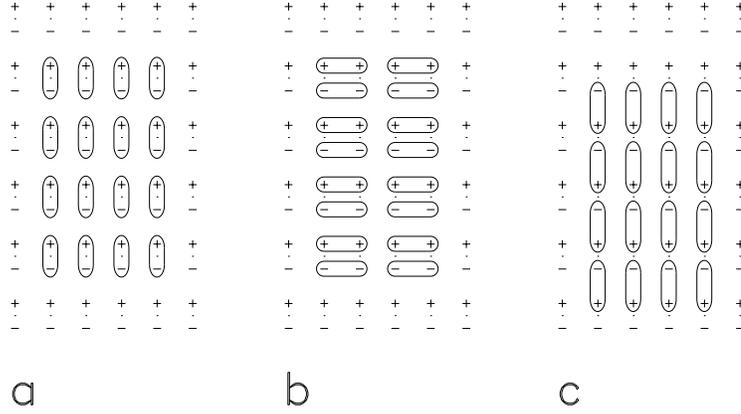


Figure 2: Possible distributions of stimuli among neurons. **a:** Stimuli of different signs, but at equal positions are combined in $\Omega_{\mathbf{r}}$ (no orientation). **b:** Stimuli of same sign, but at neighboring positions are combined in $\Omega_{\mathbf{r}}$ (oriented receptive fields, but with preference for On-cell layer or Off-cell layer). **c:** Stimuli of opposite signs, at neighboring positions, are combined in $\Omega_{\mathbf{r}}$ (Oriented receptive fields, with On-cell and Off-cell layer symmetry).

Here, we simplify the analysis considerably by assuming that the stimulus center positions are restricted to a grid in the input layers which is of the same extension as the cortical layer grid. So there are twice as many stimuli as cortical neurons. For symmetry reasons, each cortical neuron is bestmatching for two stimuli. Depending on the relation between these two stimuli, three qualitatively different map solutions can occur (Fig. 2). Assuming furthermore that

the stimuli only have an extension of 3×3 grid points, with the central position being the peak position, with amplitude $v_{peak} = 1$, and the surrounding 8 positions making up the annulus, with amplitude $v_{annulus}$, we can evaluate Eq. (4) for these three cases. The resulting analytical phase diagram for the globally stable state is depicted in Fig. 3a. Simulations of the model corroborated these results (see Fig. 3b).

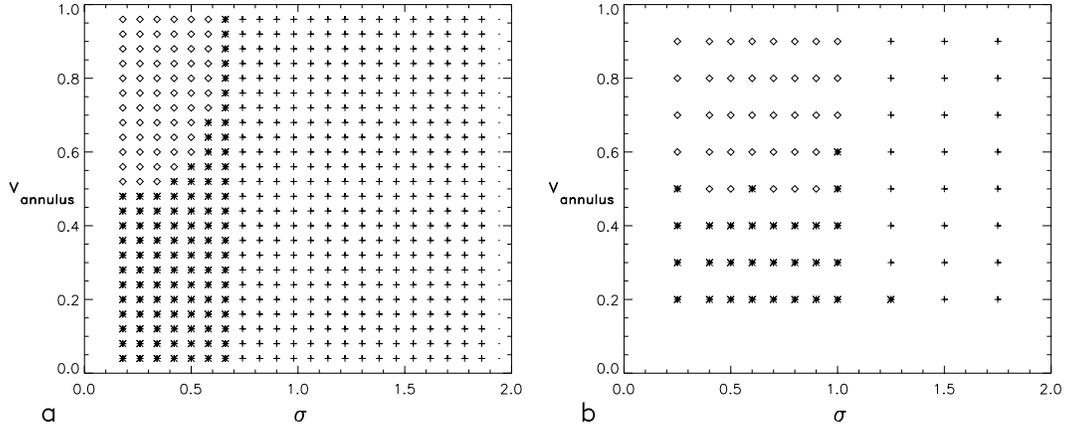


Figure 3: Phase diagrams for the different possible states of the maps. +: no orientation, \diamond : orientation, On-cell Off-cell layer asymmetry (cf. Fig. 2b), *: orientation, On-cell Off-cell layer symmetry (cf. Fig. 2c). **a:** analytical phase diagram, **b:** numerical phase diagram, with 8×8 -maps, 5×10^5 learning steps, $\epsilon = 0.2 \rightarrow 0.01$.

Simulation results for the full model

The phase diagrams for the simplified version of the model showed that the intended orientation structure occurs only at very small values of the neighborhood width σ . Consequently, we now chose such small values also for simulations of the full version of the model. In order to also be able to retain the rather small size of the stimuli, measured on the length scale of the cortical grid, we discretized the input layers with a finer grid (32×32) than the cortical grid (8×8 for our first simulations, described here). An exemplary resulting map is depicted in Fig. 4. The oriented structure of the receptive fields can clearly be seen. Neighboring receptive fields show roughly a continuous change of preferred orientations and retinal position (periodic boundary conditions!), albeit with numerous distortions due to the small value of σ enforcing this continuity.

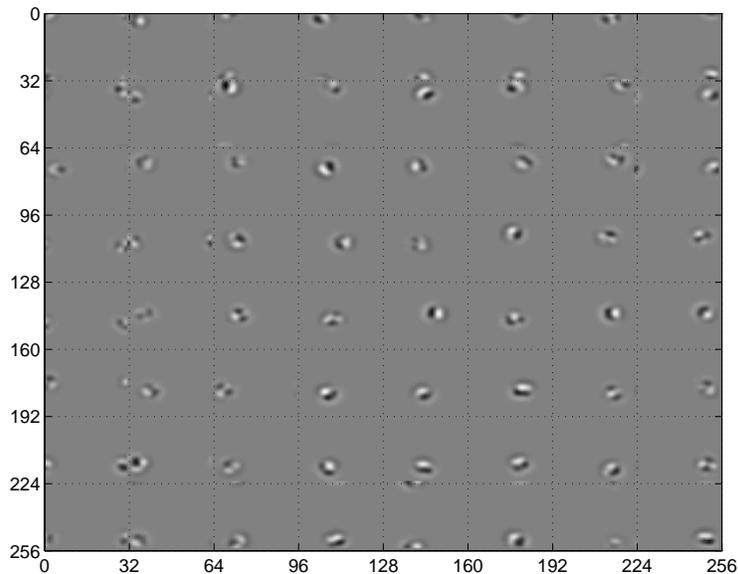


Figure 4: Final state of a 8×8 -neuron SOM-map, stimulated with difference-of-Gaussians stimuli in 32×32 input layers, width of stimuli $\sigma_1 = 1.6$, $\sigma_2 = 2.4$, $k = 0.5$, width of SOM neighborhood function $\sigma = 0.25$, 2×10^5 learning steps, $\epsilon = 0.1 \rightarrow 0.01$, periodic boundary conditions. For each neuron, the difference between On-center and Off center cell layer connection strengths is shown as a gray value image. The gray background means no connection strength, the black and white regions indicate the extension of the receptive fields.

SUMMARY AND CONCLUSIONS

The results demonstrate that the self-organization of oriented receptive fields from a competition between non-oriented stimuli in On-center and Off-center cell layers can also be achieved in the highly nonlinear SOM-model for map development, an important, as yet missing, piece in the puzzle of map formation phenomena and map model accomplishments. The results further show that a fully connected “high-dimensional” SOM model can generate maps with receptive field structures which cannot be modeled using the popular feature map approximation: In the latter class of models the emergence of novel features in the receptive fields that are not present in the stimulus set, eg., the emergence of orientation from non-oriented stimuli, is impossible.

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